

where $a_f = f'_z/f_z$. Substituting the above in the expression for $R_2(x, y, z)$ in Eq. (23) gives

$$R_2(x, y, z) = \int_{z'} h'_c(x, y, z - az', z') \oplus_{xy} [\cos(2\pi f_z(a - a_f)z') C_k(x, y) S(x, y, z')] \\ + h'_s(x, y, z - az', z') \oplus_{xy} [\sin(2\pi f_z(a - a_f)z') C_k(x, y) S(x, y, z')] dz',$$

where

$$h_c(x, y, z - az', z') = h(x, y, z - az', z') \cos(2\pi f_z(z - az') - \varphi_{kz}), \\ h_s(x, y, z - az', z') = h(x, y, z - az', z') \sin(2\pi f_z(z - az') - \varphi_{kz}).$$

Equation (23) can be now written as

$$G(x, y, z) = \int_{z'} h(x, y, z - az', z') \oplus_{xy} S(x, y, z') dz' \\ + \int_{z'} h_c(x, y, z - az', z') \oplus_{xy} [\cos(P_z(z')) \cos(P(x, y)) S(x, y, z')] dz' \\ - \int_{z'} h_s(x, y, z - az', z') \oplus_{xy} [\sin(P_z(z')) \cos(P(x, y)) S(x, y, z')] dz' \\ + \int_{z'} h(x, y, z - az', z') \oplus_{xy} [\cos(2P(x, y)) S(x, y, z')] dz',$$

where $P_z(z') = 2\pi f_z(a - a_f)z'$ and $P(x, y) = 2\pi(X_k x + Y_k y) + \varphi_k - \phi_s$. The above equation is clearly equivalent to Eq. (10).

Appendix C

The goal is to compare the result of applying $M_1(\bullet)$ and $M_2(\bullet)$ on a delta function located at $(0, 0, z_0)$ given by $\delta(x, y, z - z_0)$. Substituting $\delta(x, y, z - z_0)$ in the place of (\bullet) in Eqs. (13) and (14), we get

$$\bar{M}_{1,z_0}(x, y, z) = M_1(\delta(x, y, z - z_0)) = h_c(x, y, z), \\ \bar{M}_{2,z_0}(x, y, z) = M_2(\delta(x, y, z - z_0)) = \cos(P_z(z_0))h_c(x, y, z) - \sin(P_z(z_0))h_s(x, y, z)$$

Note that $h_c(x, y, z) = h(x, y, z) \cos(2\pi f_z z - \varphi_{kz})$, and $h_s(x, y, z) = h(x, y, z) \sin(2\pi f_z z - \varphi_{kz})$, where $h(x, y, z)$ is the widefield PSF. Hence $\bar{M}_{2,z_0}(x, y, z)$ can be written as

$$\bar{M}_{2,z_0}(x, y, z) = h(x, y, z) \cos(2\pi f_z z - \varphi_{kz} + P_z(z_0)).$$

This shows that $\bar{M}_{1,z_0}(x, y, z)$ and $\bar{M}_{2,z_0}(x, y, z)$ differ only by the phase of the cosine modulation, and hence their Fourier transforms have equal magnitude.

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